Joint Probabilistic Matching Using *m*-Best Solutions

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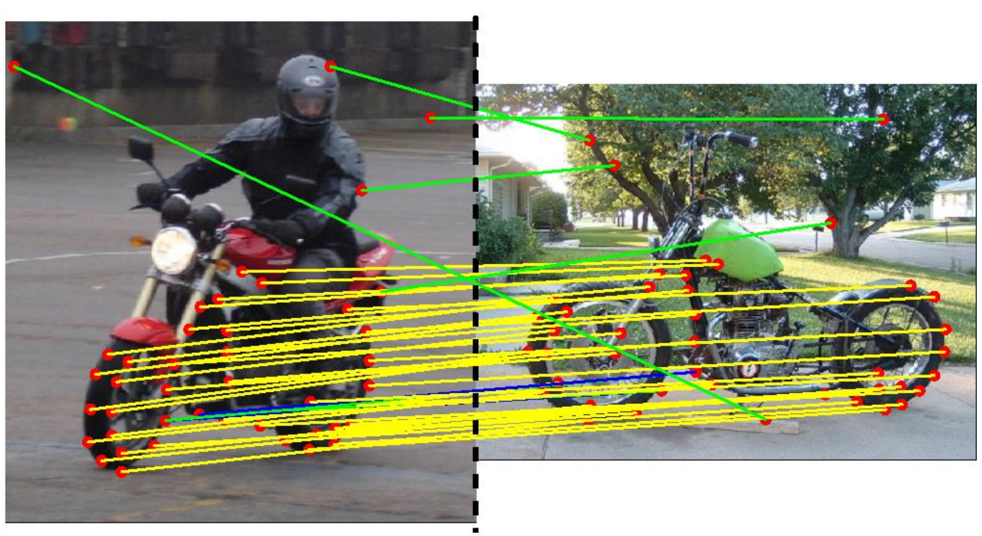
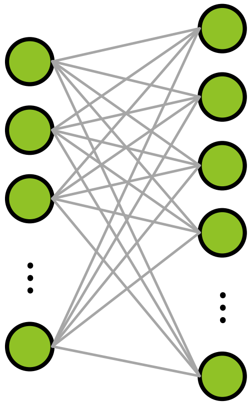
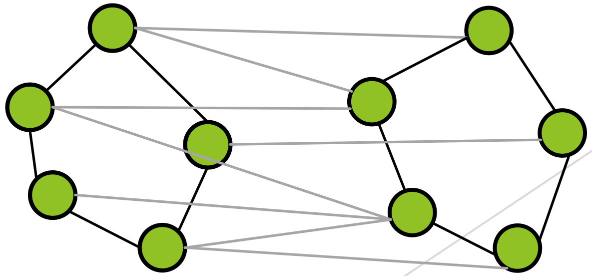
Introduction

- One-to-One Graph Matching in Computer Vision
  - Action Recognition
  - Feature Point Matching
  - Multi-Target Tracking
  - Person Re-Identification







Introduction

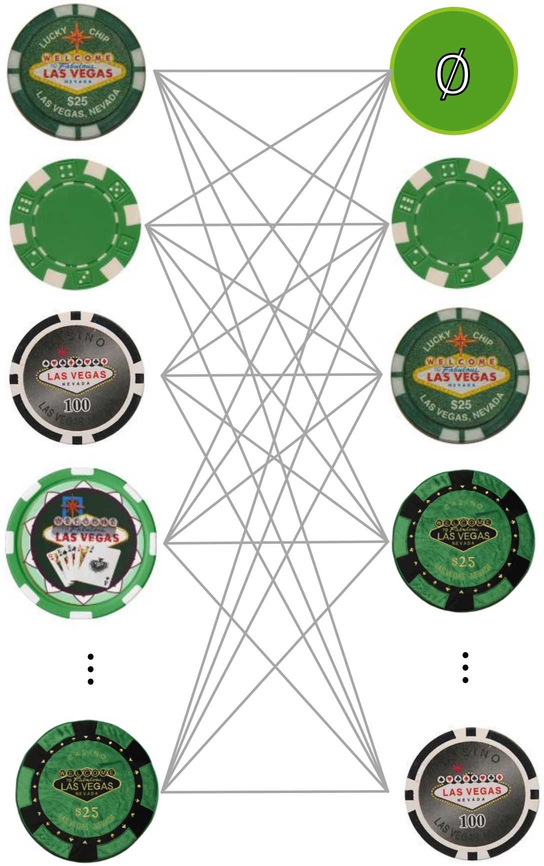
- Most existing works focus on
  - Feature and/or metric learning [Zhao *et al*., CVPR 2014, Liu *et al*., ECCV 2010]
  - Developing better solvers [Cho *et al*., ECCV 2010, Zhou & De la Torre, CVPR 2013]

- The optimal solution does not necessarily yield the correct matching assignment

- To improving the matching results, we propose
  - to consider more feasible solutions
  - a principle approach to combine the solutions

One-to-One Graph Matching

- Formulating it as a constrained binary program



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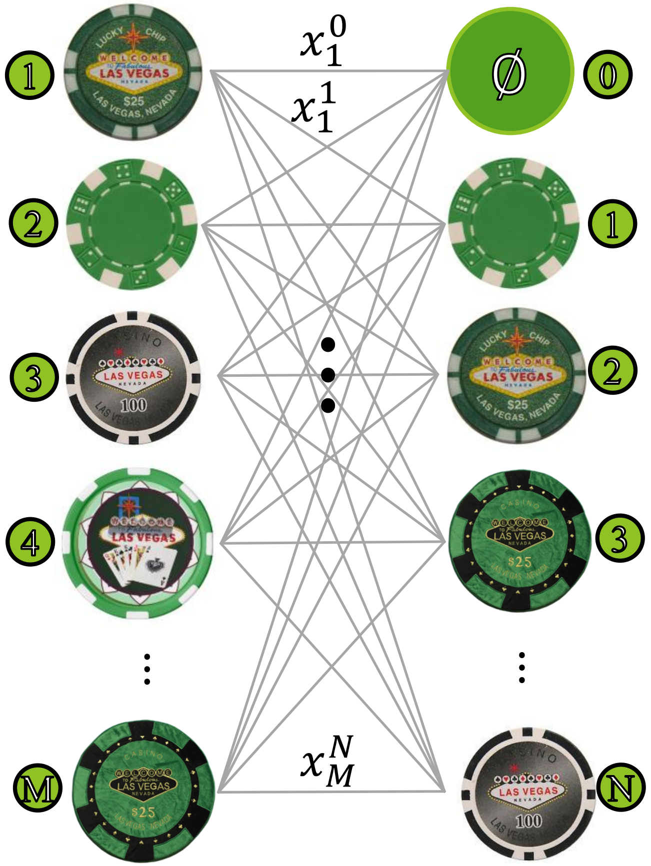

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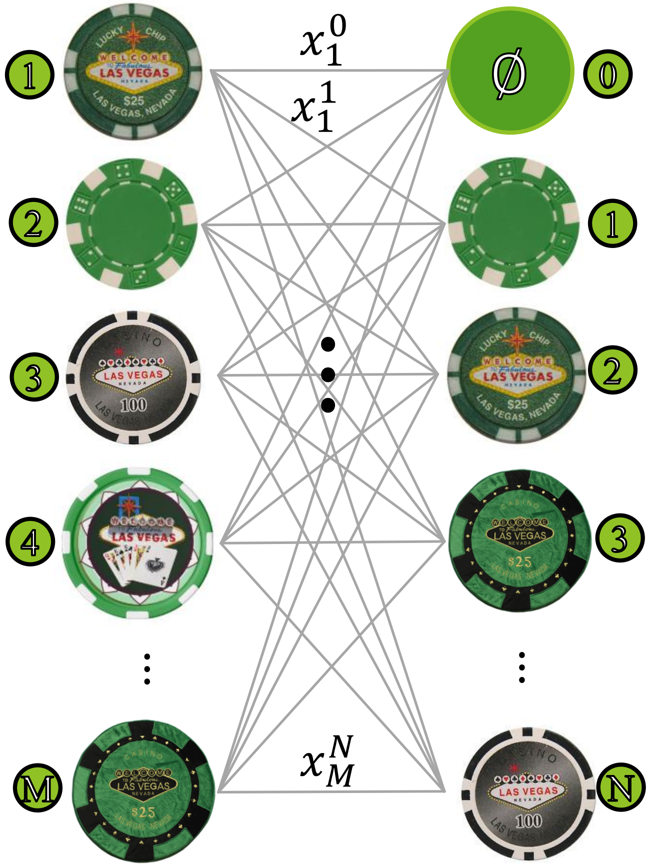

$$x_i^j = \{0,1\}$$

$$X = \left(x_1^0, x_1^1, \dots, x_i^j, \dots, x_M^N\right)^T \subseteq \mathbb{B}^{M\times(N+1)}$$

One-to-One Graph Matching

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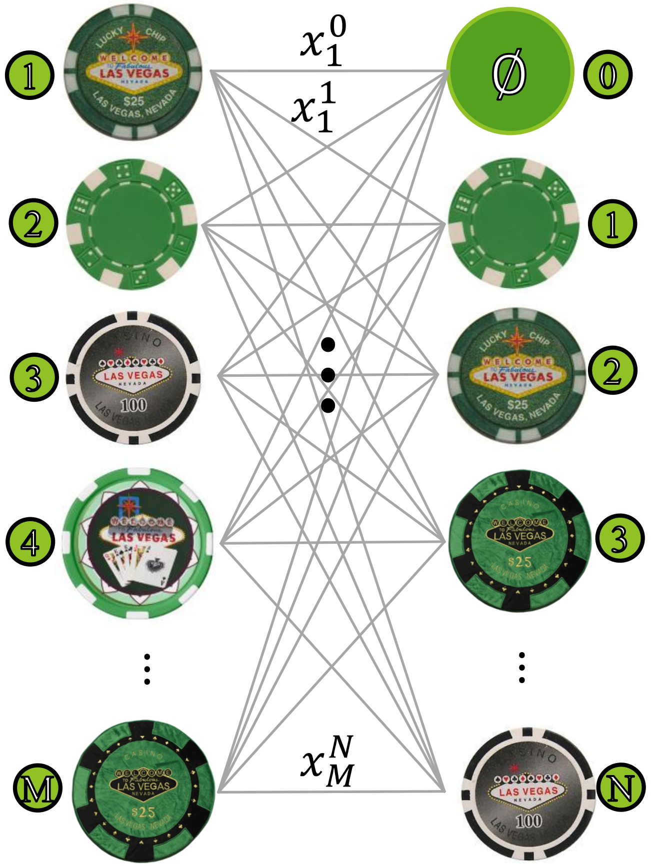

$$X^* = \underset{X \in \mathcal{X}}{\operatorname{argmin}} f(X)$$

Or

$$X^* = \underset{X \in \mathcal{X}}{\operatorname{argmax}}\, p(X)$$

where

$$\mathcal{X} = \Big\{ X = (x_i^j)_{\forall i,j} \,|\; x_i^j = \{0,1\},\ \forall j: \sum x_i^j \leq 1,\ \forall i: \sum x_i^j = 1 \Big\}$$

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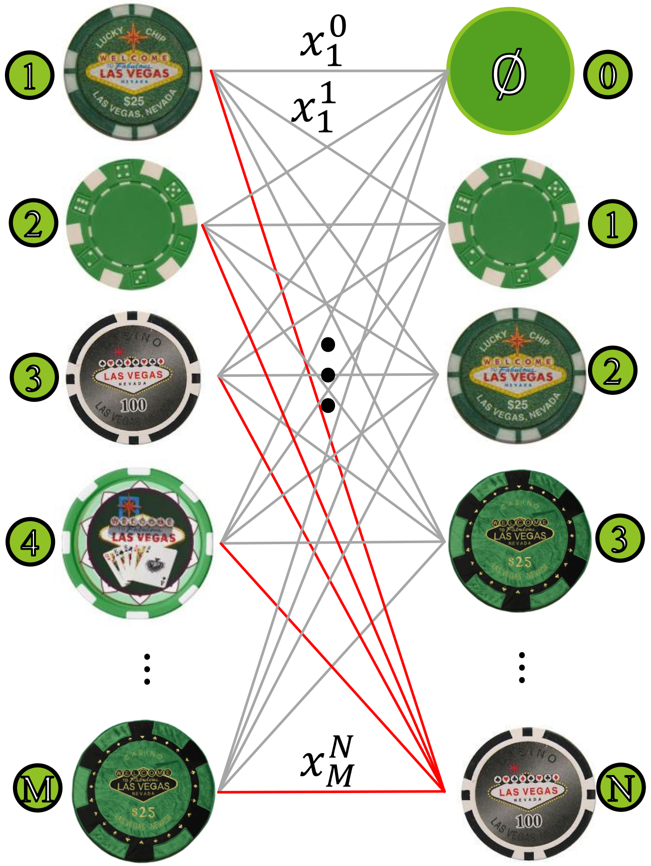

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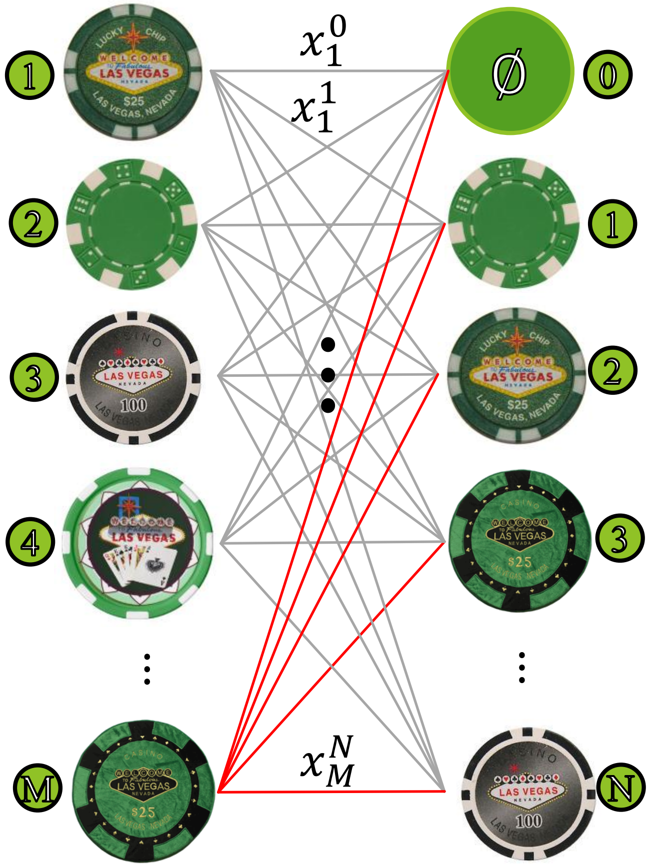

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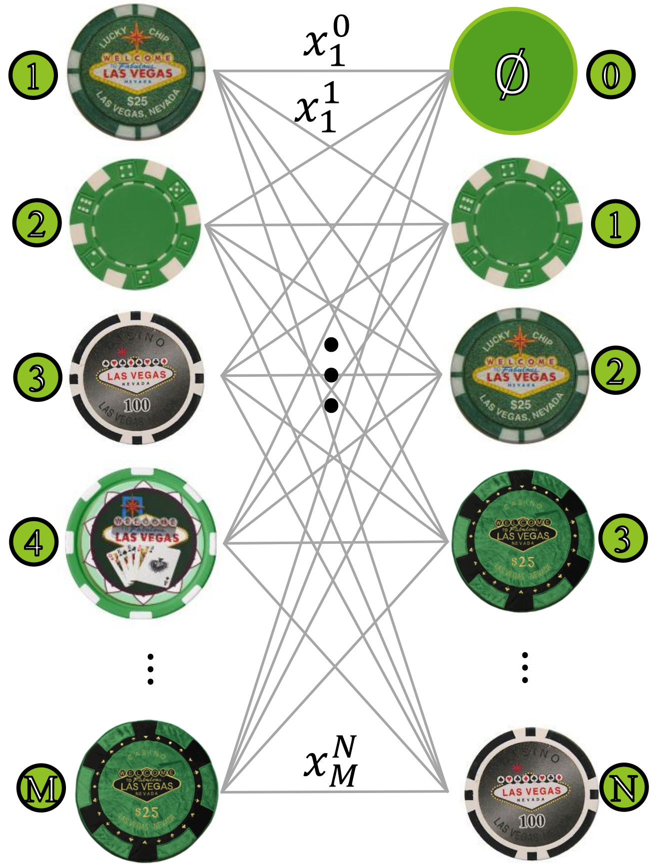

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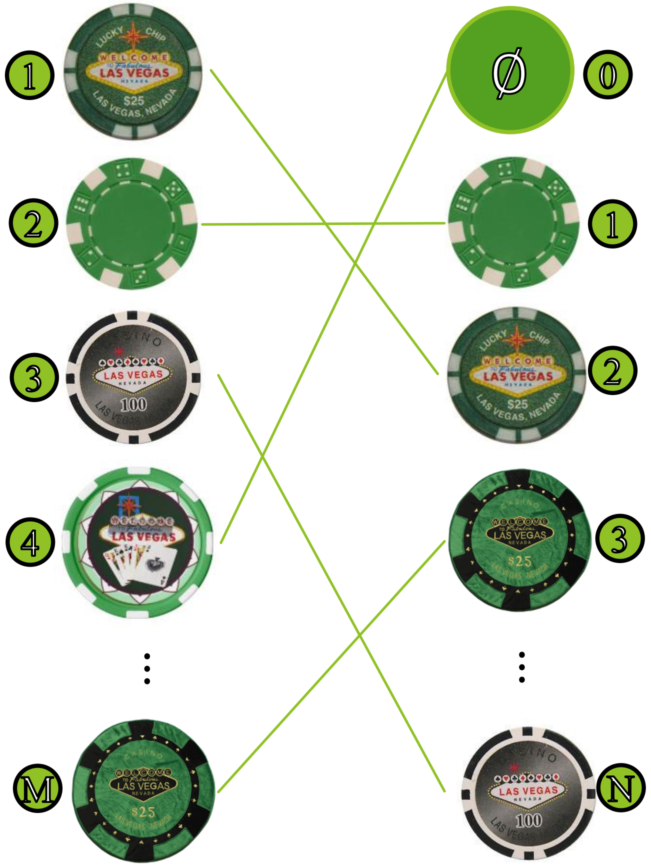

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One-to-One Graph Matching

- Examples of joint matching distribution $p(X)$ and cost $f(X)$ in different applications
  - Multi-target tracking [Zheng *et al.*, CVPR 2008] and person re-identification [Das *et al.*, ECCV 2014]

    $f(X) = C^T X$ or equivalently $p(X) \propto \prod p(x_i^j)^{x_i^j}$

  - Feature point matching [Leordeanu *et al.*, IJCV 2011]

    $f(X) = X^T Q X$

  - Stereo matching [Meltzer *et al.*, ICCV 2005] and iterative closest point [Zheng, IJCV 1994]

    higher-order constraints in addition to one-to-one constraints

Marginalization VS MAP Estimates

- In general, globally optimal solution may or may not be easily achieved.

$$X^* = \underset{X \in \mathcal{X}}{\operatorname{argmin}} f(X) \qquad X^* = \underset{X \in \mathcal{X}}{\operatorname{argmax}} p(X)$$

- Even the optimal solution does not necessarily yield the correct matching assignment

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  - Visual similarity
  - Other ambiguities in the matching space

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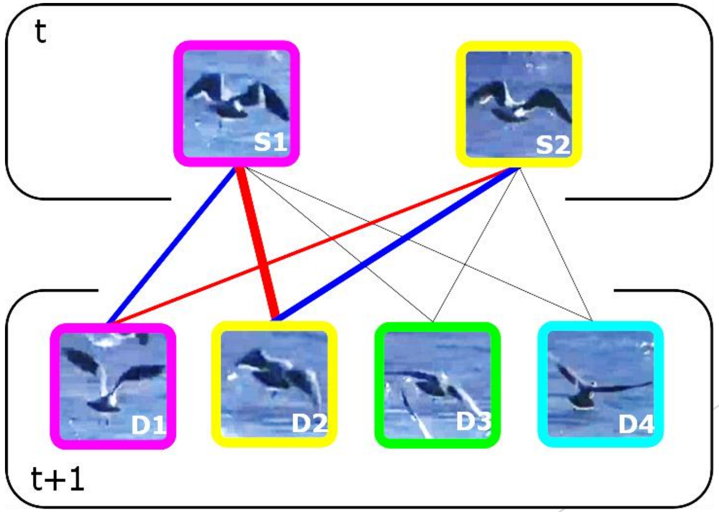

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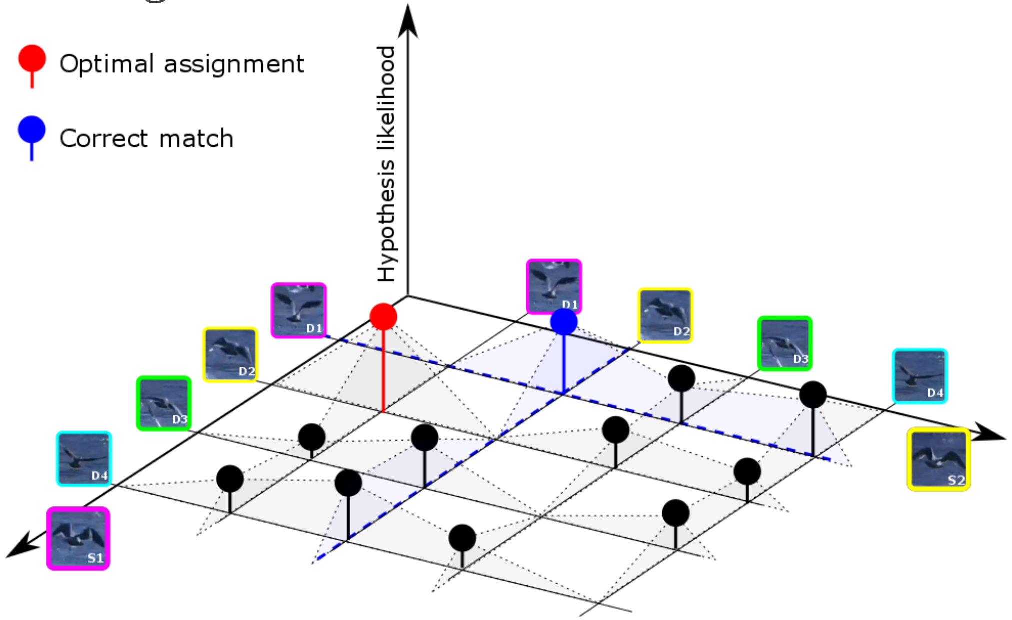

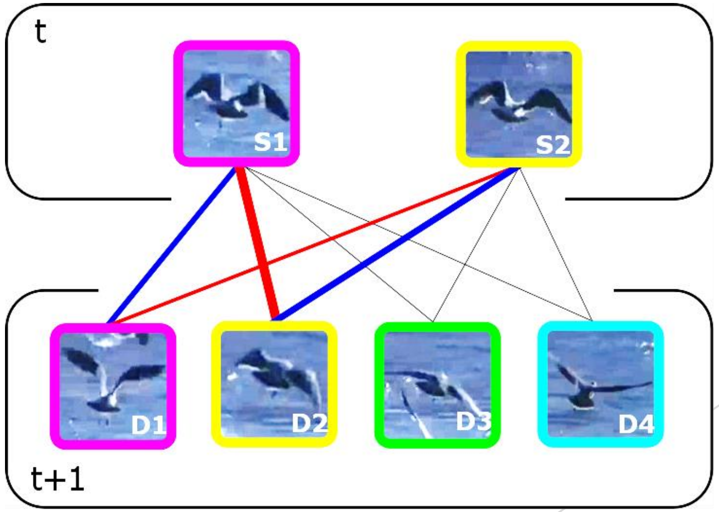

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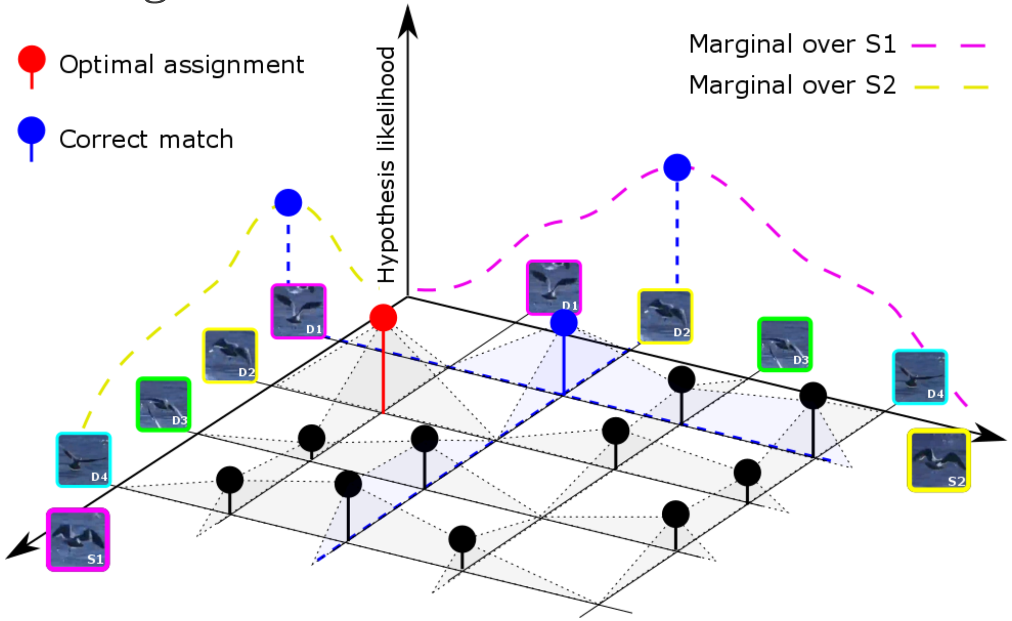
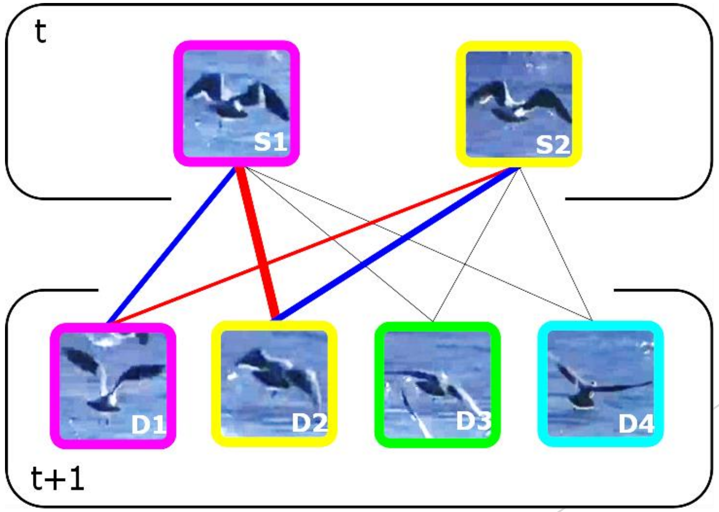




Marginalization VS MAP Estimates

Motivation to use marginalization

- ✓ Encoding the entire distribution to untangle potential ambiguities
  - ✗ MAP only considers one single value of that distribution
- ✓ Improving matching ranking due to averaging / smoothing property

Exact marginalization is NP-hard

- ✗ Requiring all feasible permutations to built the joint distribution

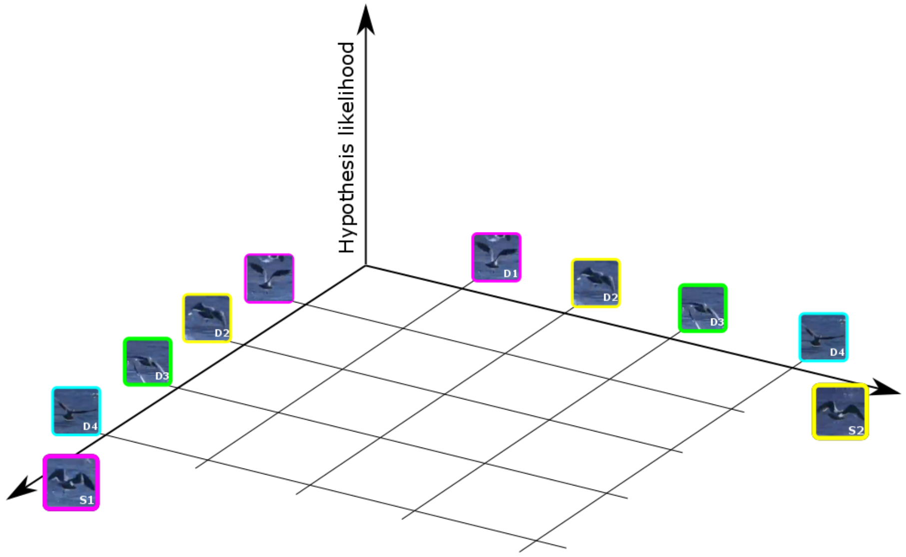
Solution

- ✓ Approximation using *m*-Best solutions

Marginalization Using *m*-Best Solutions

Marginalization by considering a fraction of the matching space

- Using m-highest joint probabilities $p(X)$/ m-lowest values for $f(X)$



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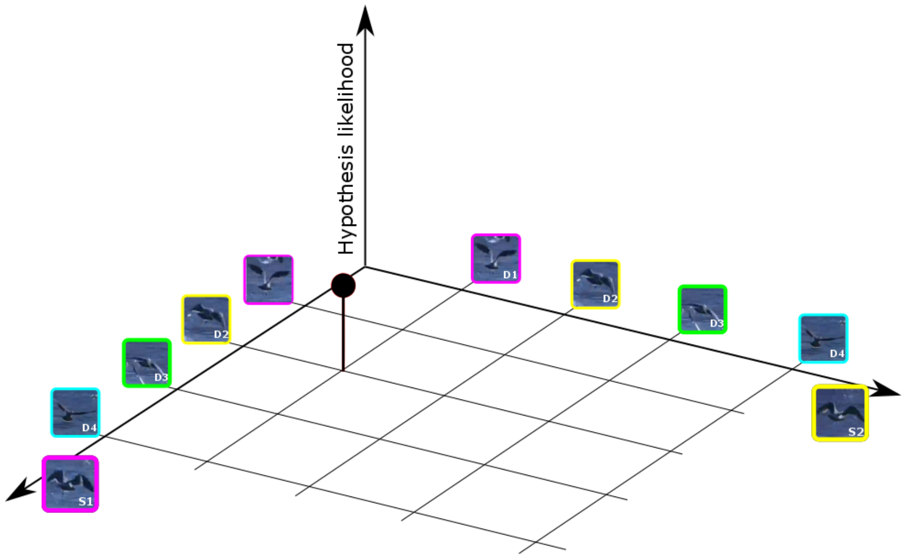

X_1^* is 1-st optimal solution

$$X^* = \underset{X \in \mathcal{X}}{\operatorname{argmax}}\, p(X)$$

$$X^* = \underset{X \in \mathcal{X}}{\operatorname{argmin}}\, f(X)$$

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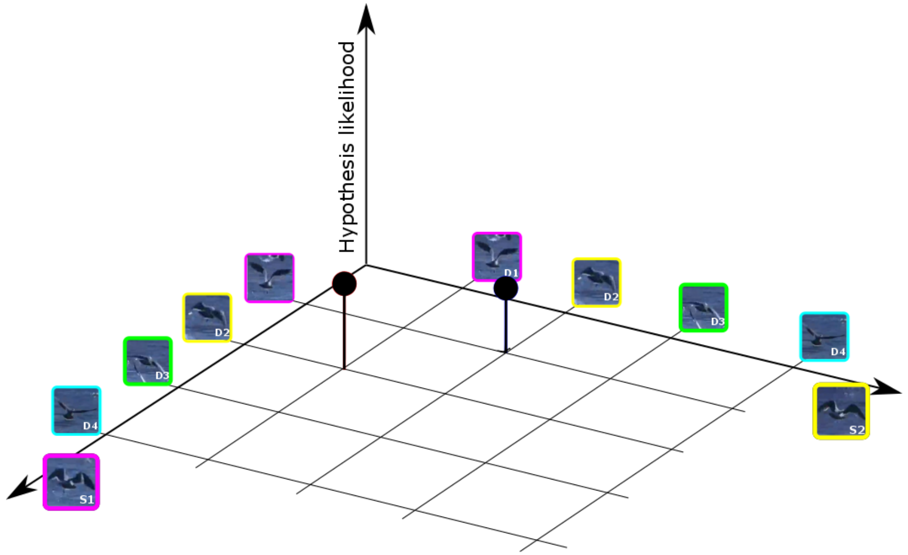

X_2^* is 2-nd optimal solution

$$X^* = \underset{X \in \mathcal{X}}{\operatorname{argmax}}\, p(X)$$

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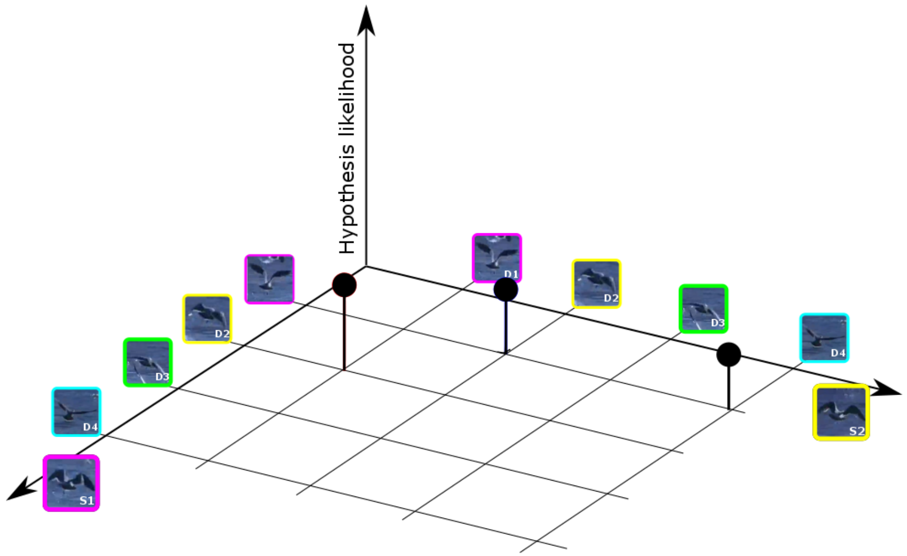

X_3^* is 3-rd optimal solution

$$X^* = \underset{X \in \mathcal{X}}{\operatorname{argmax}}\, p(X)$$

$$X^* = \underset{X \in \mathcal{X}}{\operatorname{argmin}}\, f(X)$$

Marginalization Using *m*-Best Solutions

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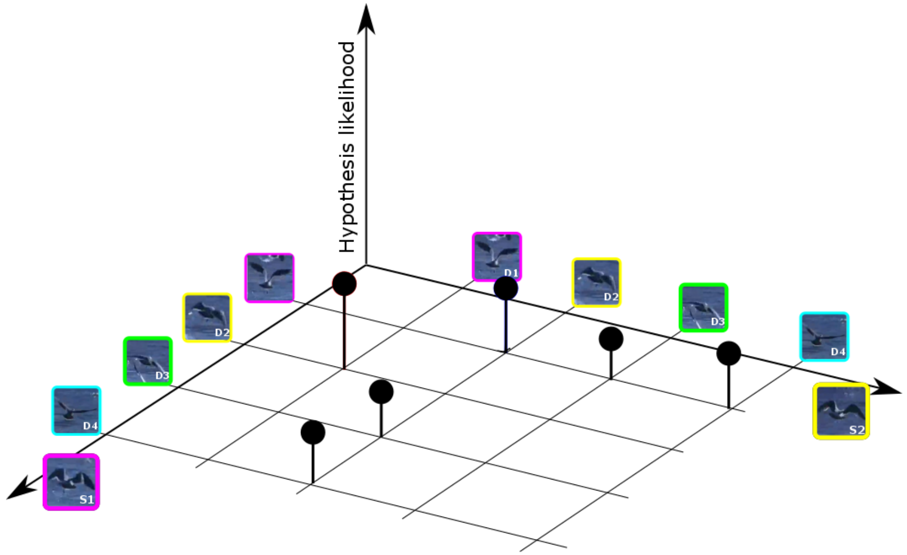

X_k^* is k-th optimal solution

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Marginalization Using m-Best Solutions

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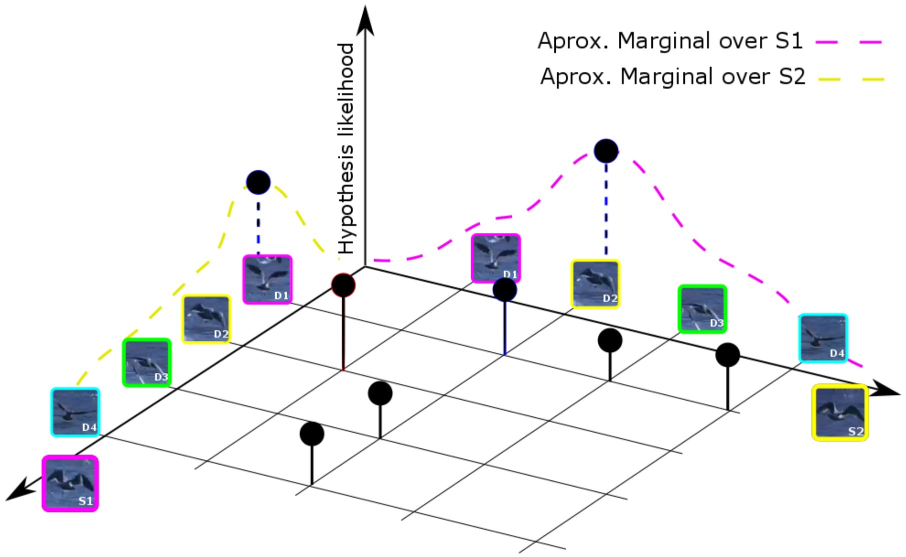

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Marginalization Using *m*-Best Solutions

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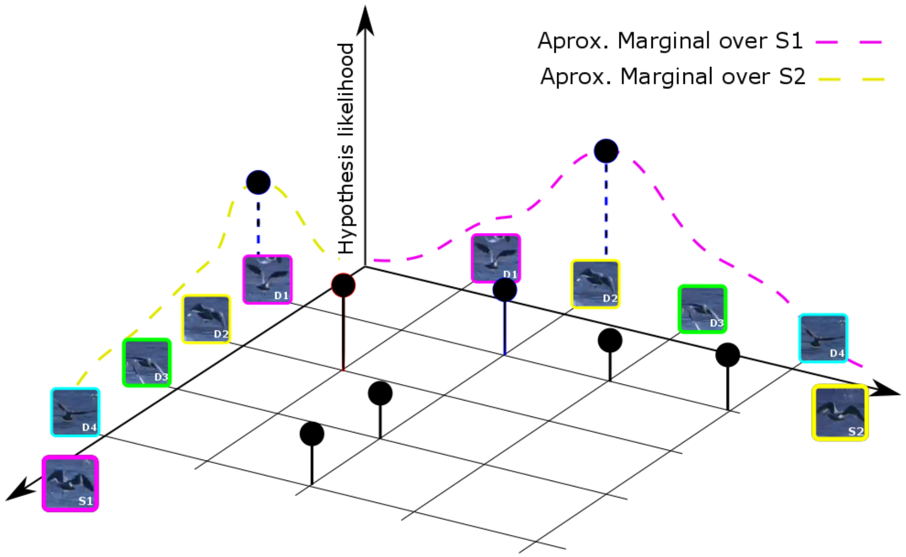

X_k^* is *k*-th optimal solution

$$X^* = \underset{X \in \mathcal{X}}{\operatorname{argmax}}\, p(X)$$

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- Approximation error bound decreases exponentially by increasing number of solutions [Rezatofighi *et al.* , ICCV 2015]

Computing the *m*-Best Solutions

Naïve exclusion strategy

$$X_1^* = \operatorname{argmin} f(X)$$
$$AX \leq B$$

Computing the *m*-Best Solutions

Naïve exclusion strategy

$$X_2^* = \operatorname{argmin} f(X)$$
$$AX \leq B$$
$$\langle X, X_1^* \rangle \leq \|X_1^*\|_1 - 1$$

Computing the *m*-Best Solutions

Naïve exclusion strategy

$$
\begin{aligned}
X_3^* = \operatorname{argmin} f(X) \\
AX \leq B \\
\langle X, X_1^* \rangle \leq \|X_1^*\|_1 - 1 \\
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\end{aligned}
$$

Computing the *m*-Best Solutions

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\begin{aligned}
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\langle X, X_2^* \rangle \leq \|X_2^*\|_1 - 1 \\
\vdots \\
\langle X, X_{k-1}^* \rangle \leq \|X_{k-1}^*\|_1 - 1
\end{aligned}
$$

Computing the *m*-Best Solutions

Naïve exclusion strategy

- ✓ General approach
- ✗ Impractical for large values of *m*

$$X_k^* = \text{argmin } f(X)$$
$$AX \leq B$$
$$\acute{A}X \leq \acute{B}$$

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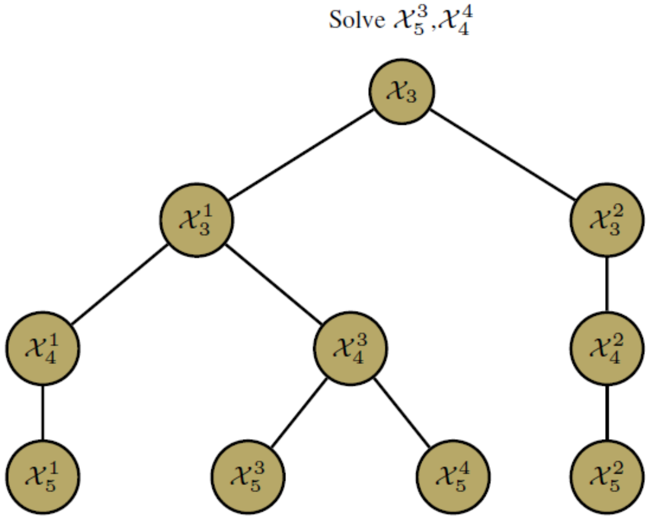
Binary Tree Partitioning

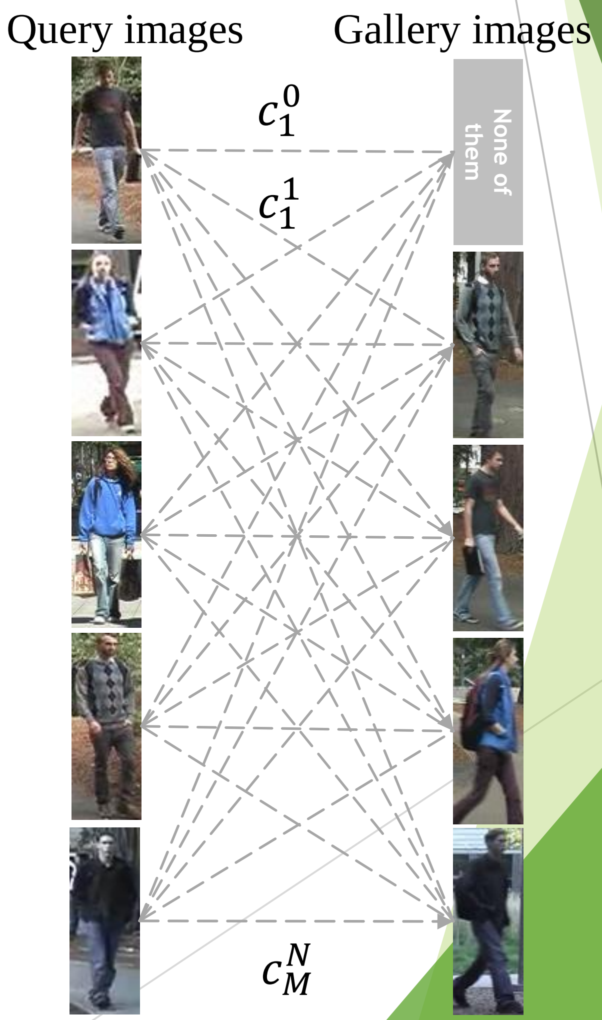
Partitioning the space into a set of disjoint subspaces [Rezatofighi *et al.*, ICCV 2015]

- ✓ Efficient approach
- ✘ Not a good strategy for weak solvers



Experimental Results

Person Re-Identification



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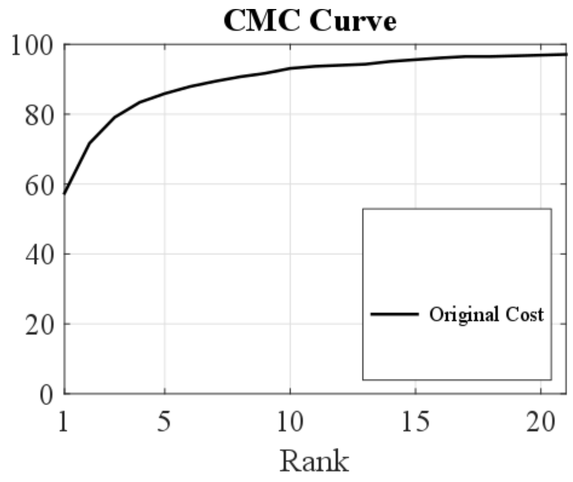
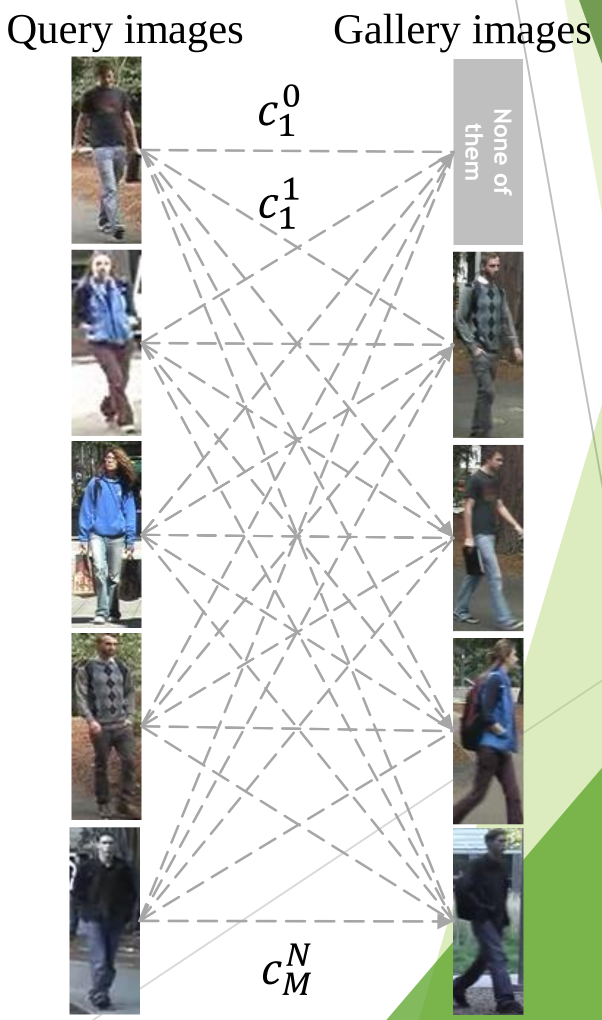


Original Assignment Costs

Gallery images (columns) / Query images (rows)

c_1^0	c_1^1	$\cdots$	c_1^N
c_2^0	c_2^1	$\cdots$	$\cdots$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
c_M^0	c_M^1	$\cdots$	c_M^N



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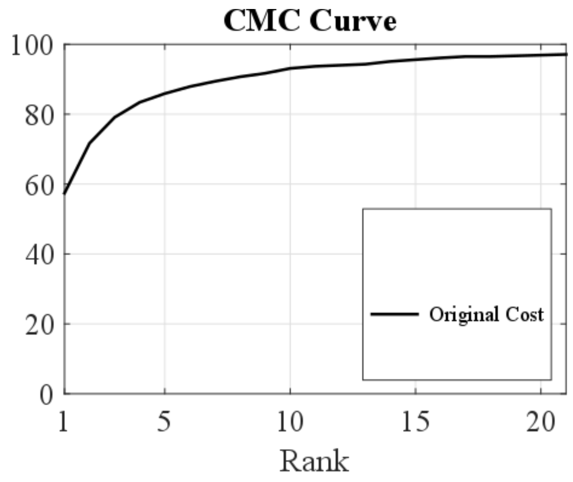
$$X^* = \underset{X \in \mathcal{X}}{\operatorname{argmin}}\, C^T X$$



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m-bst

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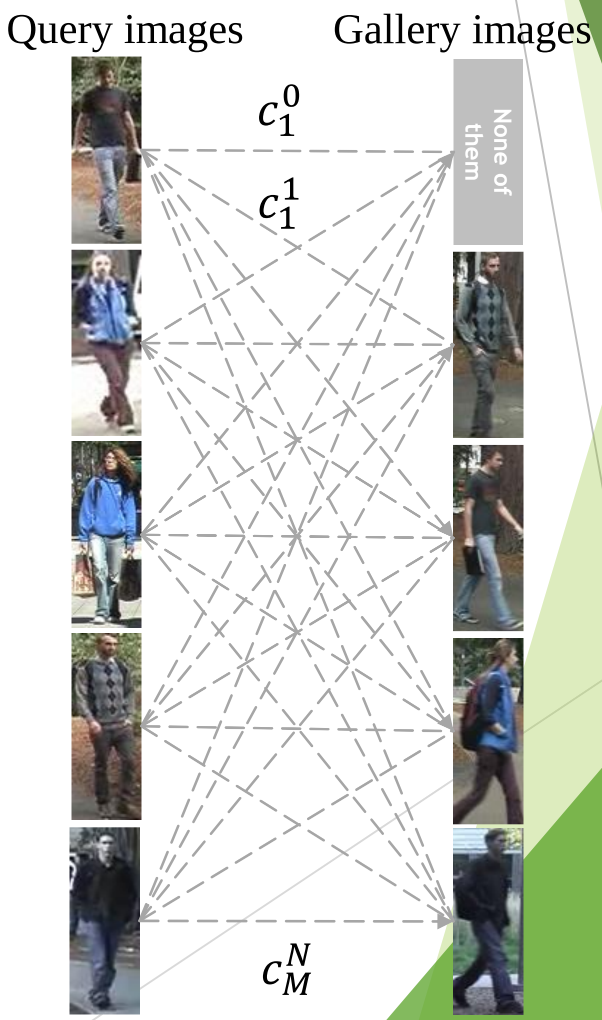
c_2^0	c_2^1	$\cdots$	$\cdots$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
c_M^0	c_M^1	$\cdots$	c_M^N

***m*-best Marginalized Costs**



Experimental Results

Person Re-Identification

- ✓ Ranking is improved

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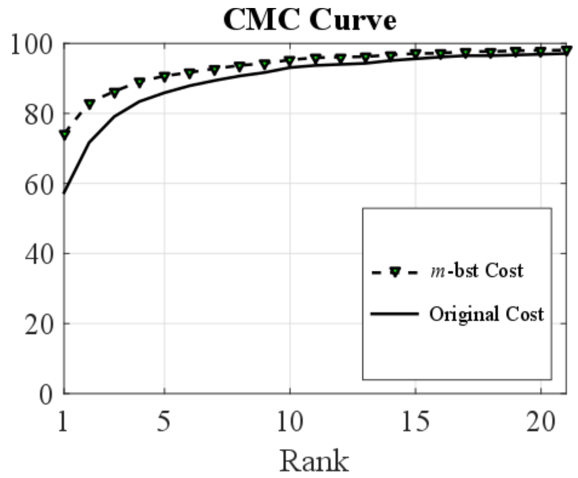

Original Assignment Costs

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c_1^0	c_1^1	$\cdots$	c_1^N
c_2^0	c_2^1	$\cdots$	$\cdots$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
c_M^0	c_M^1	$\cdots$	c_M^N

m-bst

Gallery images (columns) / Query images (rows)

c_1^0	c_1^1	$\cdots$	c_1^N
c_2^0	c_2^1	$\cdots$	$\cdots$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
c_M^0	c_M^1	$\cdots$	c_M^N

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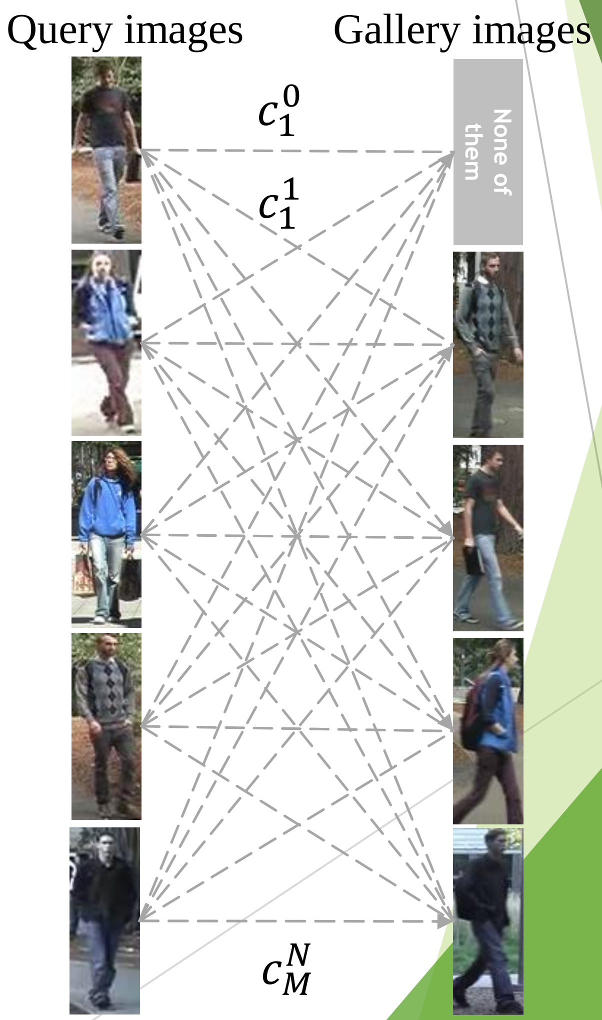

Experimental Results

Person Re-Identification

FT [Das *et al.*, ECCV 2014] **AvgF** [Paisitkriangkrai *et al.*, CVPR 2015]

Dataset (Size)	Method (m=100)	Recognition rate %			Time (Sec.)
		Rank-1	Rank-2	Rank-5	
RAiD (20×20)	FT	74.0	82.0	96.0	
	*m*bst-FT	**85.0**	**99.0**	**100.0**	1.6
iLIDS (59×59)	AvgF	51.9	60.7	72.4	
	*m*bst-AvgF	**54.7**	**63.6**	**75.4**	15.4
VIPeR (316×316)	AvgF	44.9	58.3	76.3	
	*m*bst-AvgF	**50.5**	**63.0**	**78.0**	201.9

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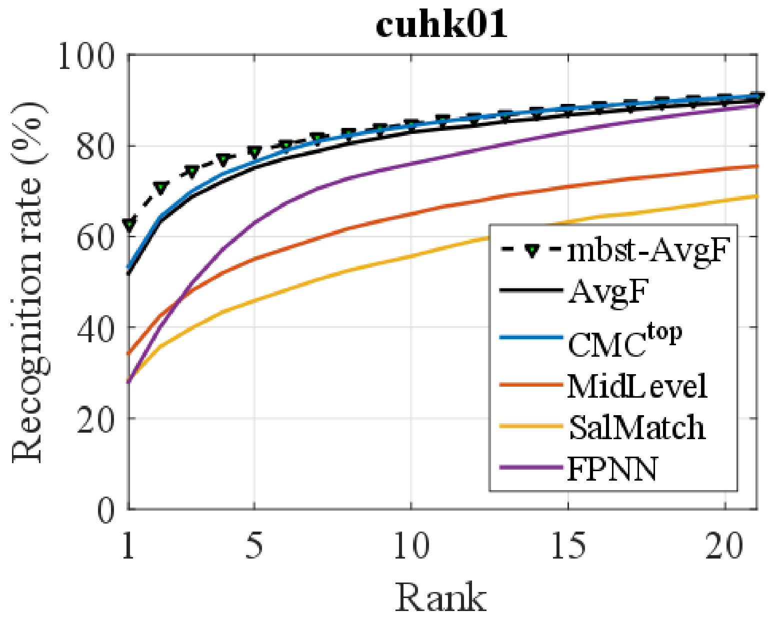
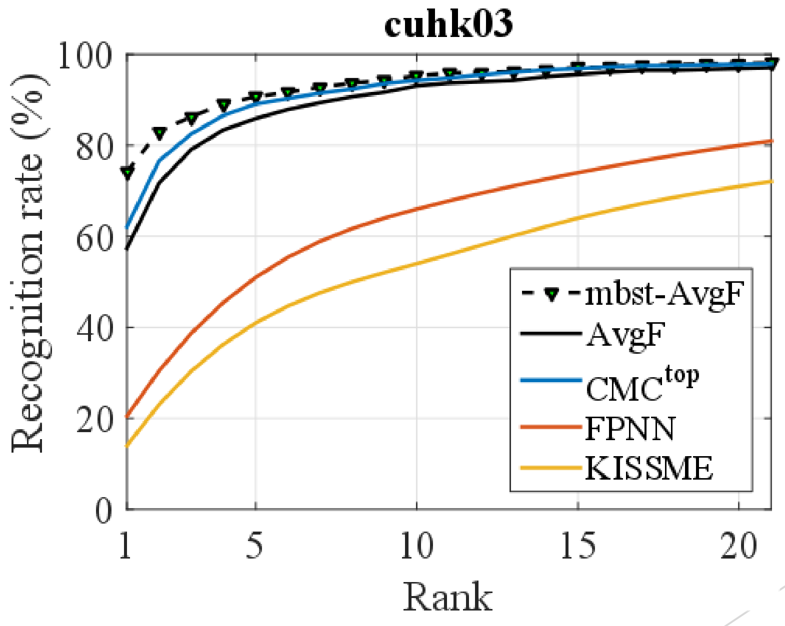
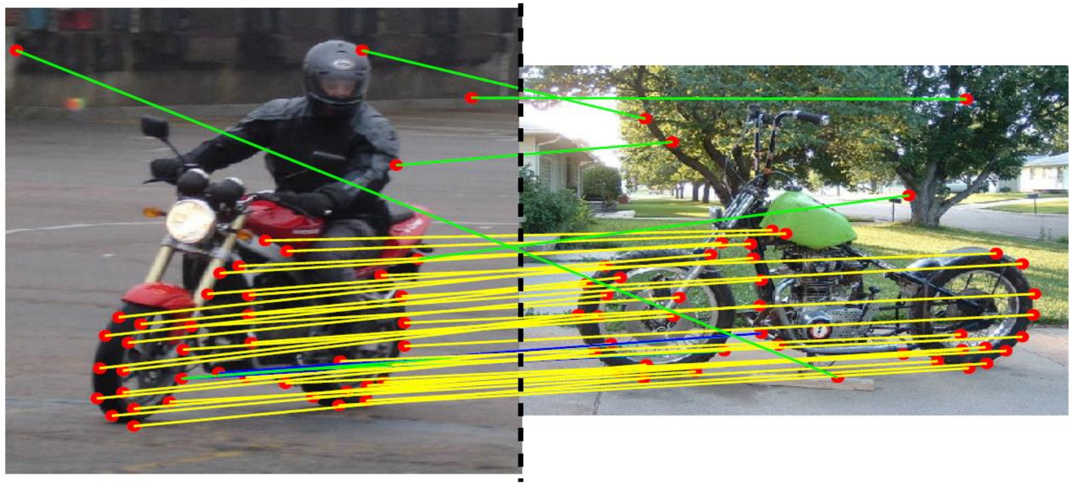




Experimental Results

Feature Matching

$$X^* = \underset{X \in \mathcal{X}}{\operatorname{argmax}} X^T K X$$



Matching PASCAL VOC dataset
[Leordeanu *et al.*, IJCV 2011]

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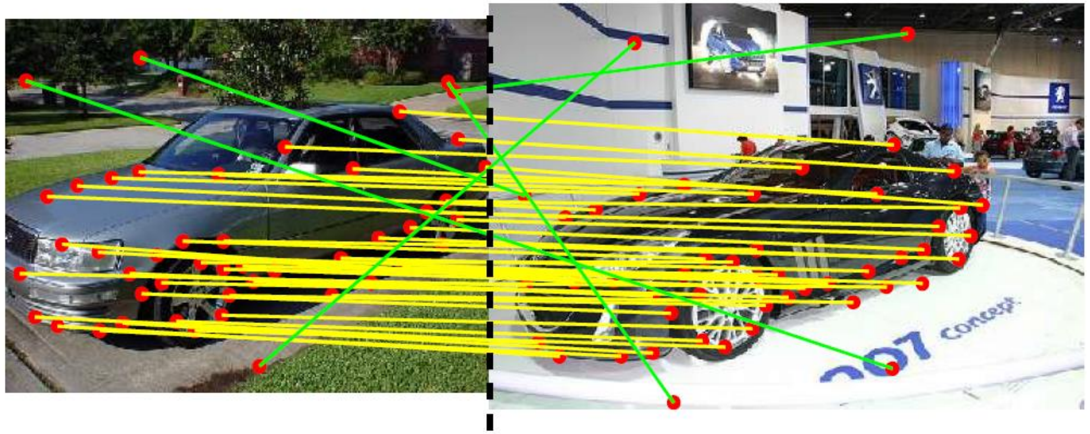

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Experimental Results

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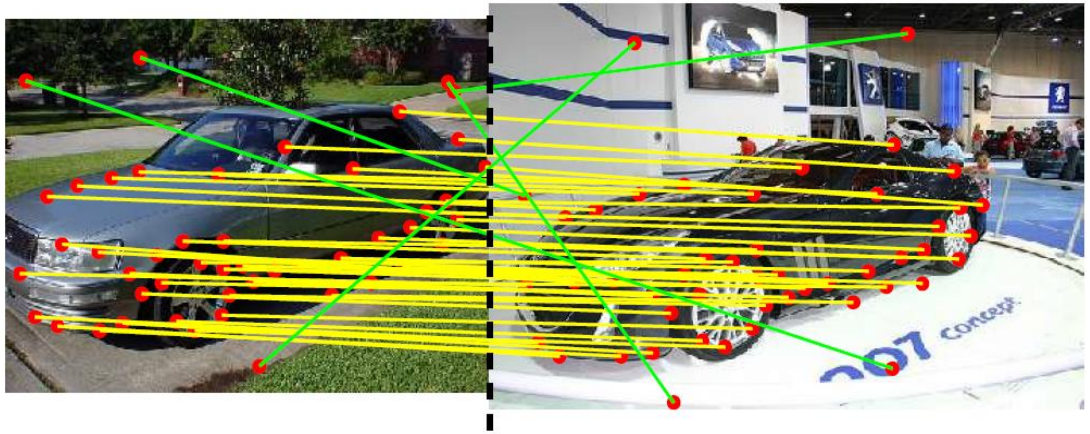
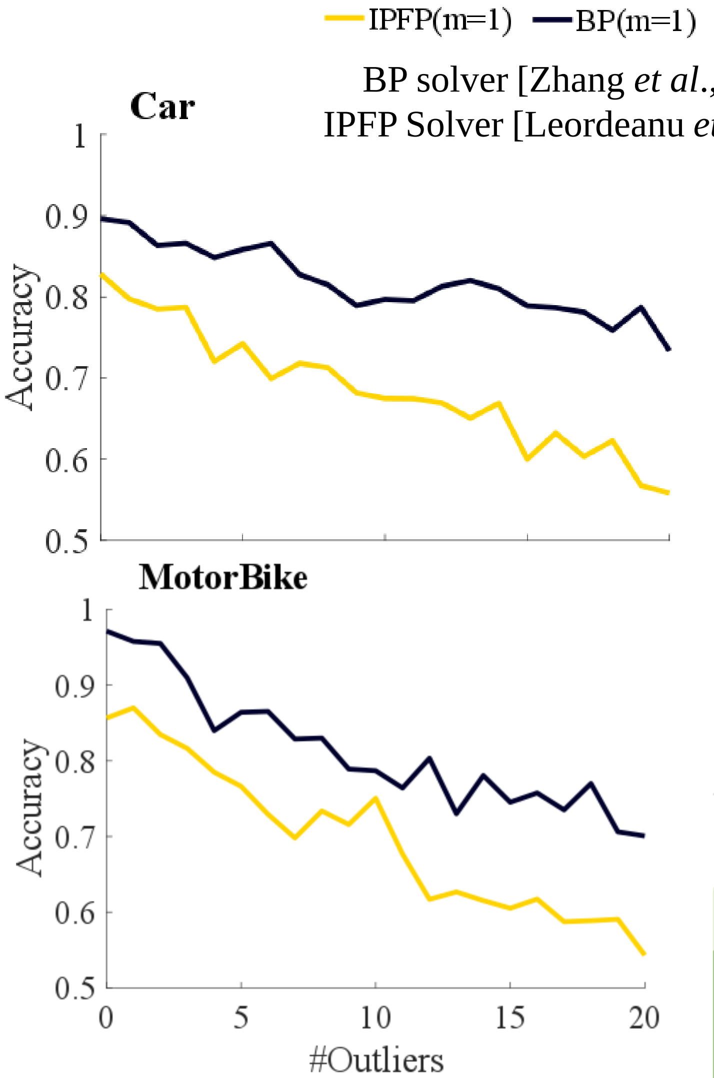
$$X \in \mathcal{X}$$



Matching PASCAL VOC dataset
[Leordeanu *et al.*, IJCV 2011]

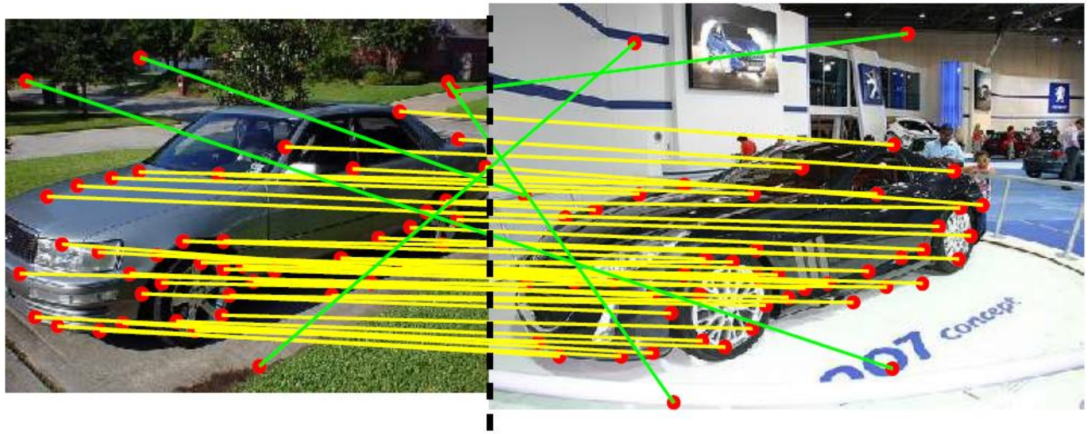


BP solver [Zhang *et al.*, CVPR 2016]
IPFP Solver [Leordeanu *et al.*, IJCV 2011]

Experimental Results

Feature Matching

$$X^* = \underset{X \in \mathcal{X}}{\operatorname{argmax}} \; X^T K X$$



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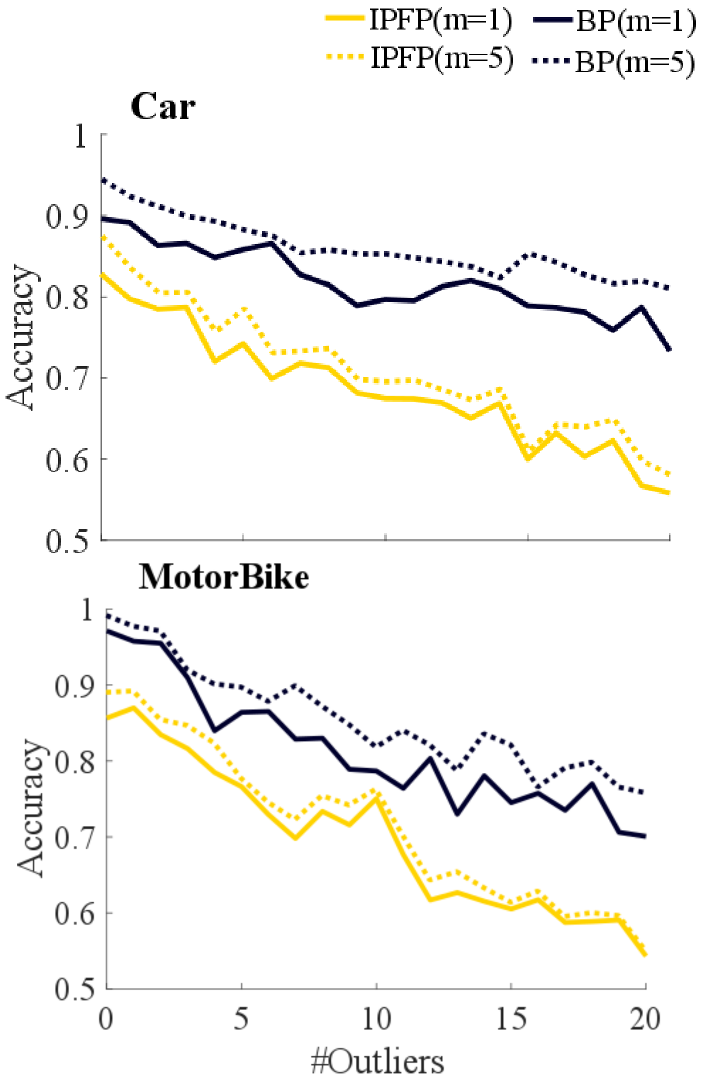
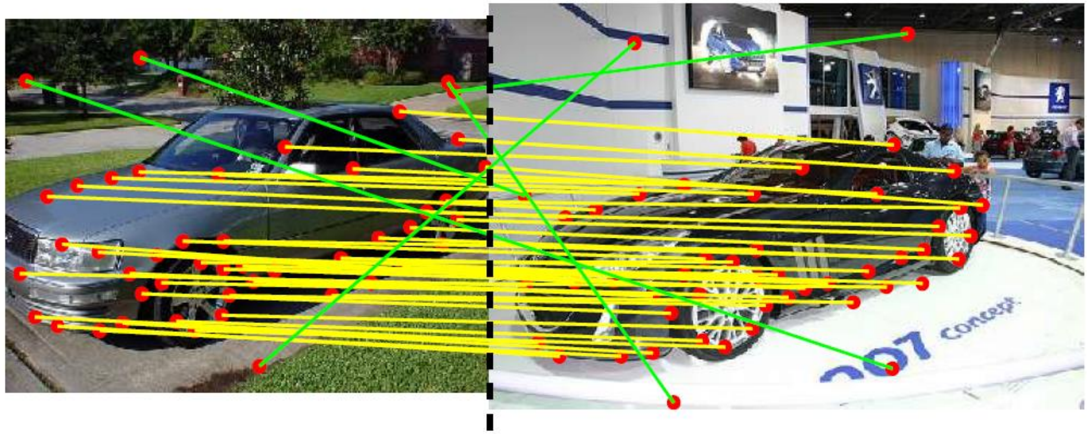


Experimental Results

Feature Matching

$$X^* = \operatorname{argmax}_{X \in \mathcal{X}} X^T K X$$



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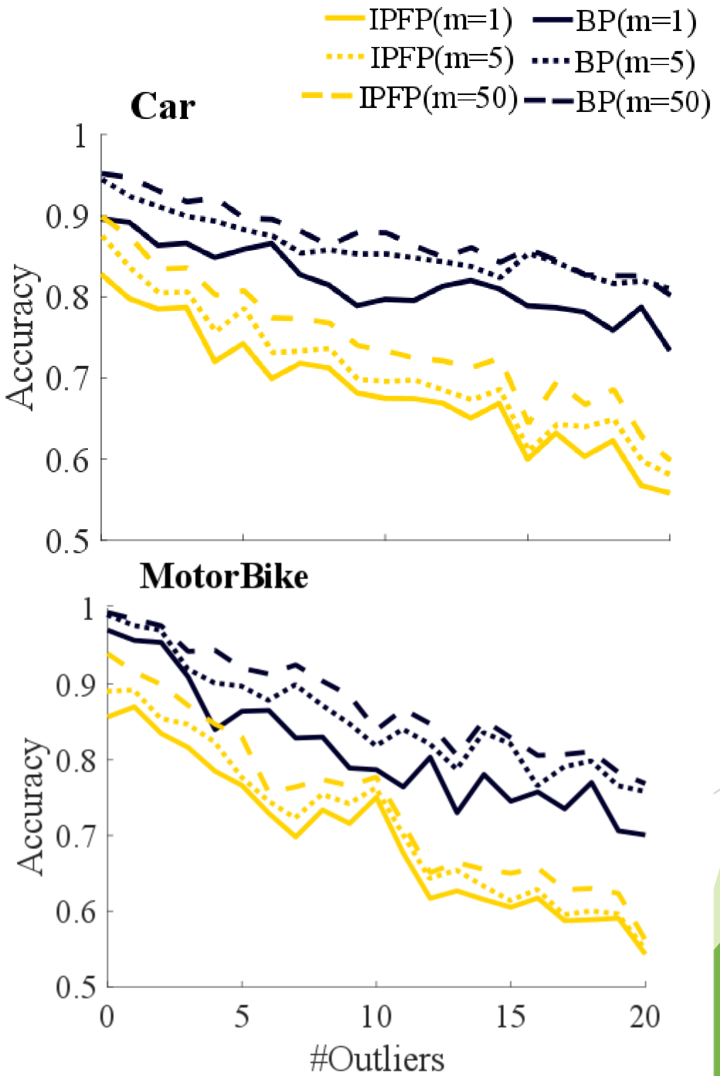
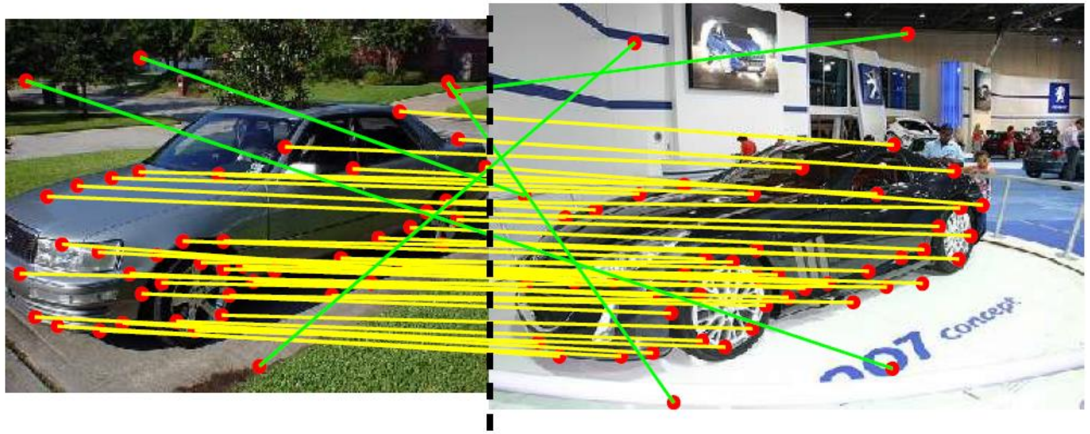


Experimental Results

Feature Matching

$$X^* = \underset{X \in \mathcal{X}}{\operatorname{argmax}}\, X^T K X$$



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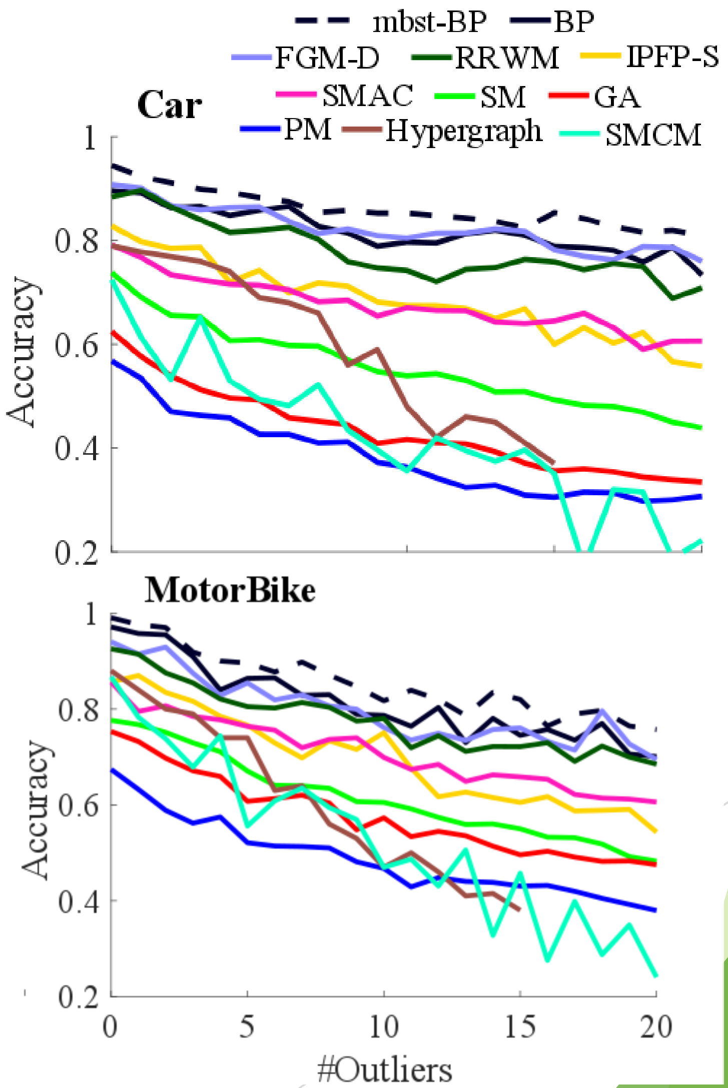

Discussion & Conclusion

Limitations

- One-to-One constraint is no longer guaranteed by marginalization
- Requires computational overhead to calculate *m* solutions

Conclusion

- Graph matching by approximated marginals using *m*-best solutions instead of MAP
- A generic approach applicable to similar problems
- Marginalization improves matching accuracy and ranking

Take-home message

- Do not rely on a single solution, explore more solutions

Future work

- Exploring further applications with arbitrary cost functions

Thank you

Visit our poster

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Code will be available